Abacus Primary School

Maths Calculation Policy

(Mental and Written)

Written by: K. Hodkin and H. Head

on: September 2023

Future Review date: December 2024

Intent:

At Abacus Primary School, we understand that a secure understanding of the four operations (addition, subtraction, multiplication and division) is essential for children to progress with their calculation knowledge and thrive across all areas of Mathematics.

Curriculum Drivers

For a child to be secure in their understanding of Mathematics, they must be able to **communicate** their methods of calculations; this could take the form of explaining their process or applying their reasoning skills. A CPA (concrete, pictorial, abstract) approach will ensure **engagement** with a hands-on approach when appropriate. This will then allow the pupils to be **independent** learners as they apply the learnt calculation skills. A well-planned approach to teaching calculations will support children's **wellbeing** as they are given the chance to be successful. This policy with support the teaching and learning of both mental and written calculations.

Implementation:

The purpose of this policy

- To develop a positive attitude and enthusiasm towards mathematics by ensuring that activities are rich and enjoyable experiences which enrich the mathematical experiences of all learners.
- To ensure that mental calculation and written strategies are complementary strategies as in all methods there is an element of mental processing.
- Children make use of diagrams and informal notes (jottings) to help record steps when using mental methods that generate more information than can be kept in their heads to support/extend the development of more fluent and sophisticated mental strategies.
- To develop children's ability to calculate, solve problems, to reason, to think logically, and to work systematically and accurately by offering plenty of opportunities to use and apply their mathematical skills.
- To ensure a consistent and progressive approach exists within the school to secure good to outstanding progress in written calculations and use of manipulatives (hands-on materials).
- For children to reflect upon which method to use to solve a problem and ask questions such as 'Can I do this in my head?', 'Can I do this in my head or do I need equipment to help me?', 'Do I need to use a written method?' then 'Is my answer sensible?'
- For children to be able to clearly explain methods of recording/representation and justify why their answers are correct using sound mathematical vocabulary and universal symbols (strong speaking and listening opportunities underpin good mathematics teaching).

This policy contains the key procedures that are to be taught throughout the school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement. Children should use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

Mental Calculations

The ability to calculate in your head is an important part of mathematics. It is also an essential part of coping with society's demands and managing everyday events. The overall aim of this policy is that when children leave our school, they have a secure knowledge of number facts and are able to solve problems mentally, selecting an efficient strategy from a range of known approaches.

The 'fundamentals' such as number bonds and multiplication tables are stressed and problem solving plays a central role in learners' mathematical development, while motivational activities help to develop inquiring minds.

In EYFS and Key Stage 1, classes will use the NCETM Mastering Number programme to support the teaching and retention of basic mental calculations/number skills.

Written calculations

"Accurate calculations and careful presentation give pupils the ability to spot important and interesting patterns of number, as well as errors that need to be corrected. Calculation methods and presentation rules are procedural knowledge that need to be taught and rehearsed to automaticity." **Research review series: mathematics** - 25 May 2021

Using and Applying Calculation Knowledge

Before children move onto the next stage in calculation, it is important that their skills are broadened through their use and application in a range of contexts (including money, time and other measures).

When are children ready for written calculations?

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division

- Do they know the 2, 3, 4, 5 and 10 time table?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

For further guidance and support on how to use a CPA approach when teaching mental and written calculations, please see Appendix A. This document shares the specific Early Years Foundation Stage Reform 2021 and the National Curriculum Mathematics for the four operations and a break down of progressive strategies for each operation.

How will this policy be monitored and reported?

This policy will be monitored by the Mathematics subject leader. It will be shared with staff and will be available on the school website.

When it will next be reviewed.

This policy is due to be reviewed December 2024.

Does this policy need to be read in conjunction with any other policy?

- Mathematic Policy

This will give you more information about resources, inclusion, assessment and how the subject is taught and planned for.

Calculation Policy

Appendix A

Below are the expectations set out by the Early Years Foundation Stage Reform 2021 and National Curriculum Mathematics Programme of Study for each year group and how they apply to calculating of the four operations.

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1170108/EYFS_framework_from_September_2023.pdf

Mathematics programmes of study: key stages 1 and 2 (publishing.service.gov.uk)

Early Years

Children at the expected level of development will:

- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Year 1

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9.
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

Pupils should be taught to:

- solve problems with addition and subtraction:
    - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
    - applying their increasing knowledge of mental and written methods
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3

Pupils should be taught to:

- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Year 4

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Year 5

Pupils should be taught to:

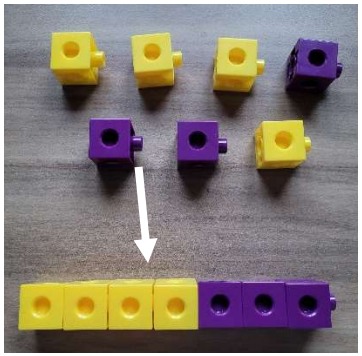
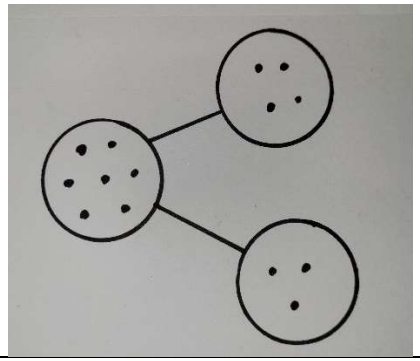
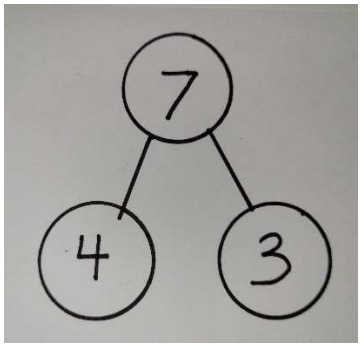
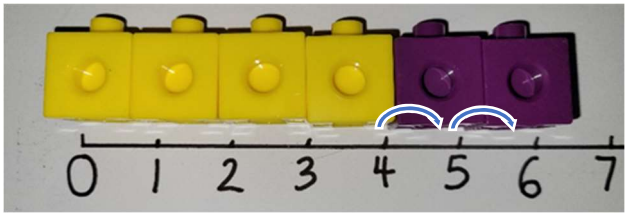
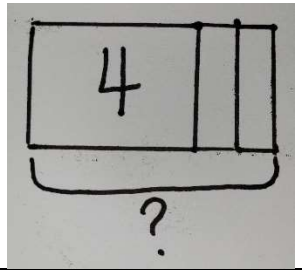
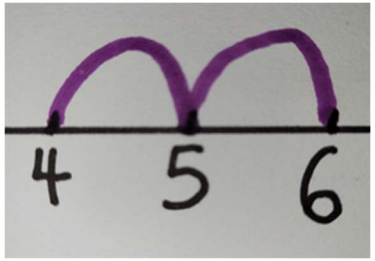
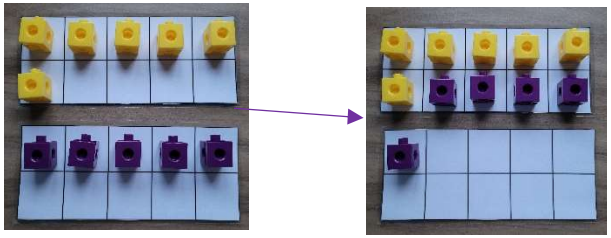
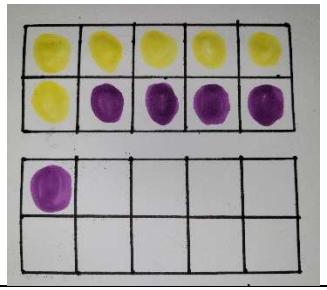
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 6

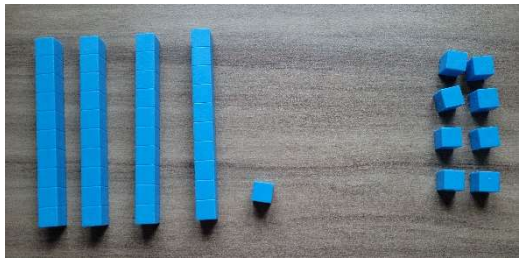
Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

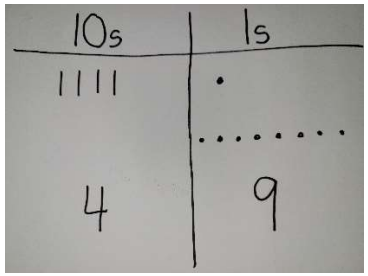
Addition

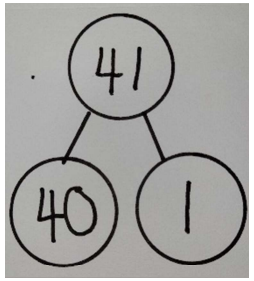
Concrete	Pictorial	Abstract
Combining two parts to make a whole Use a range of resources e.g. multi-link, teddy bears. 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	Numbers in part whole 
Counting on using a number line Use cubes or Numicon 	A bar model to encourage the child to count on rather than count all. 	Number line 
Regrouping to make 10 Using ten frames and counters/cubes or using Numicon or Rekenreks. 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality. 6+ _ = 11 6 + 5 = 5 + _ 6 + 5 = _ + 4

TO + O using base 10
Continue to develop understanding of partitioning and place value.



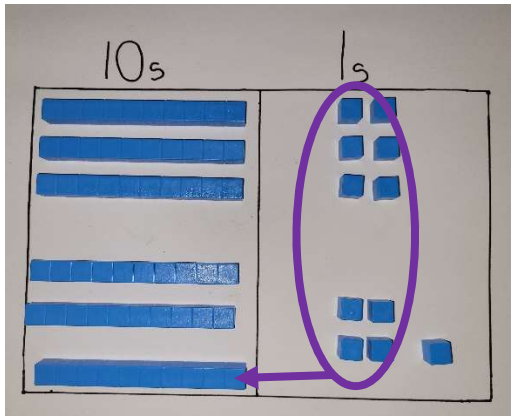
Children to represent the base 10 e.g. lines for tens and dots for ones.



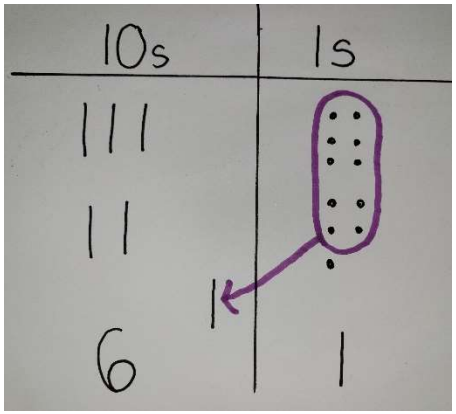


1 + 8 = 9
40 + 9 = 49

TO + TO using base 10.



6 **1**



Looking for ways to make 10.

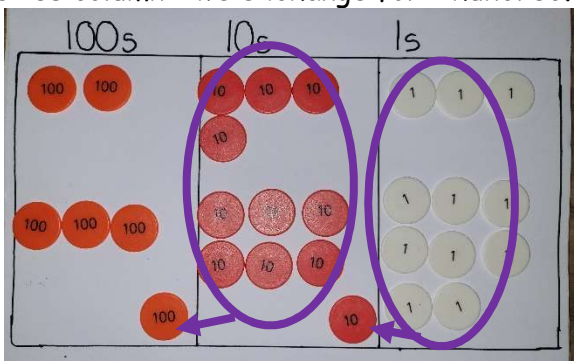
36 + 25 = *30+20 = 50*
5 + 5= 10
1 5 *50 + 10 + 1 = 61*

Formal Method
(From Year 3)

	3	6
+	2	5
	6	1
	1	

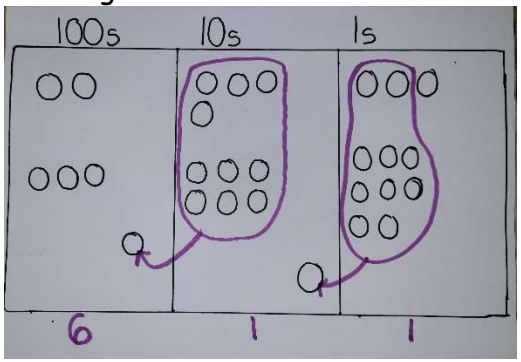
Use of base 10/ place value counters to add HTO + TO, HTO + HTO etc.

When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column – we exchange for 1 hundred.



6 1 1

Children to represent the counters in a place value chart, circling when they make an exchange.

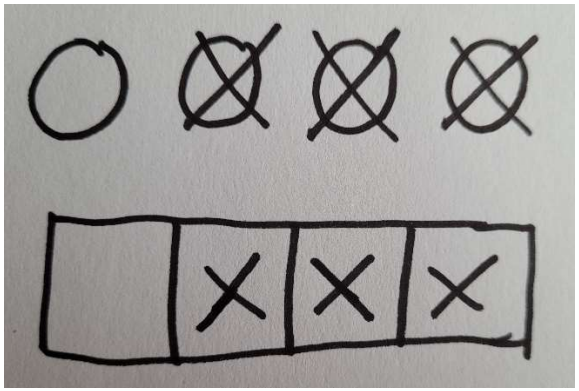
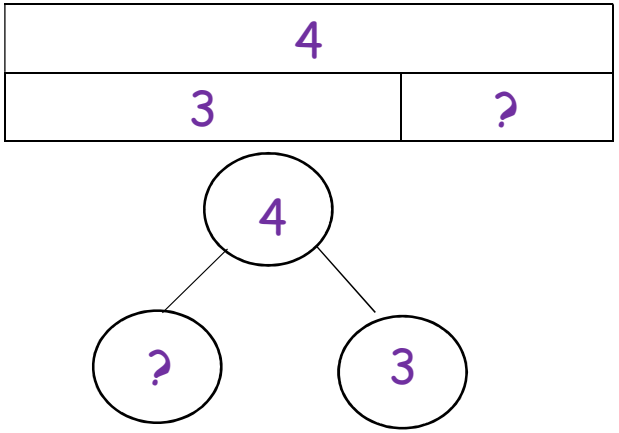
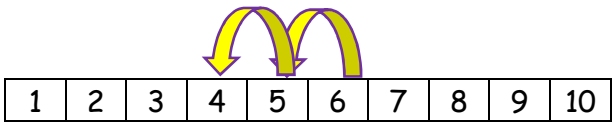
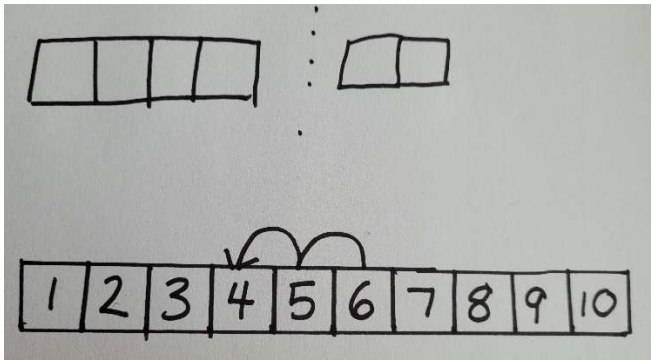
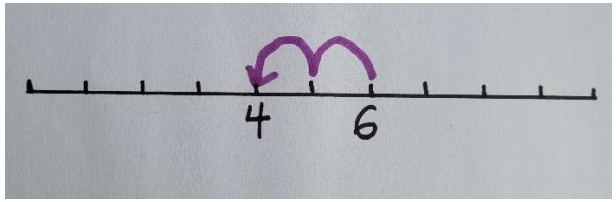


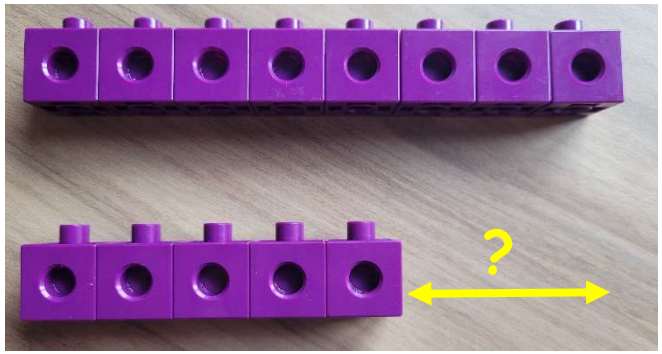
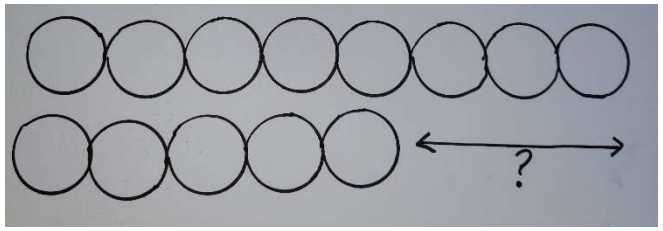
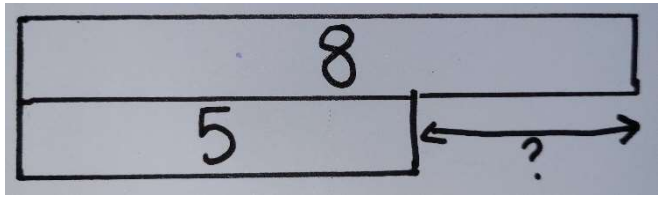
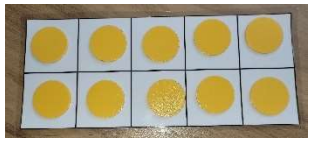
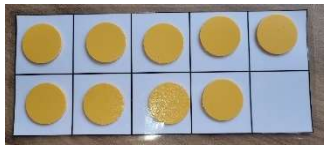
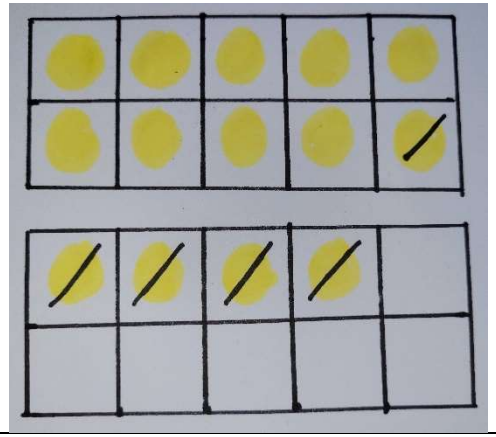
$$\begin{array}{r} 2\ 4\ 3 \\ +\ 3\ 6\ 8 \\ \hline 6\ 1\ 1 \\ \hline 1\ \ 1 \end{array}$$

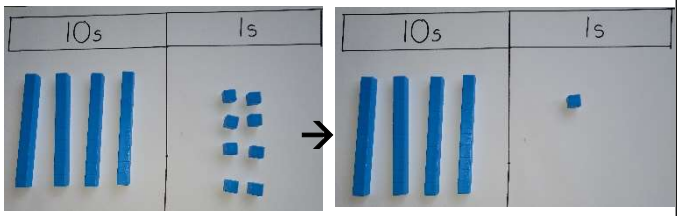
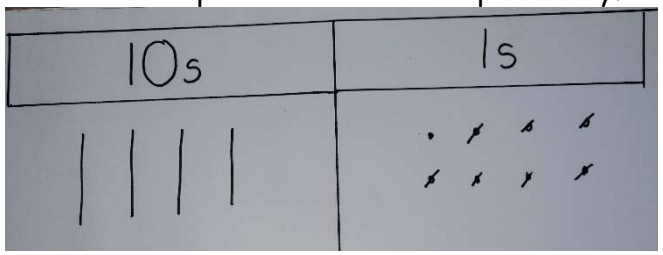
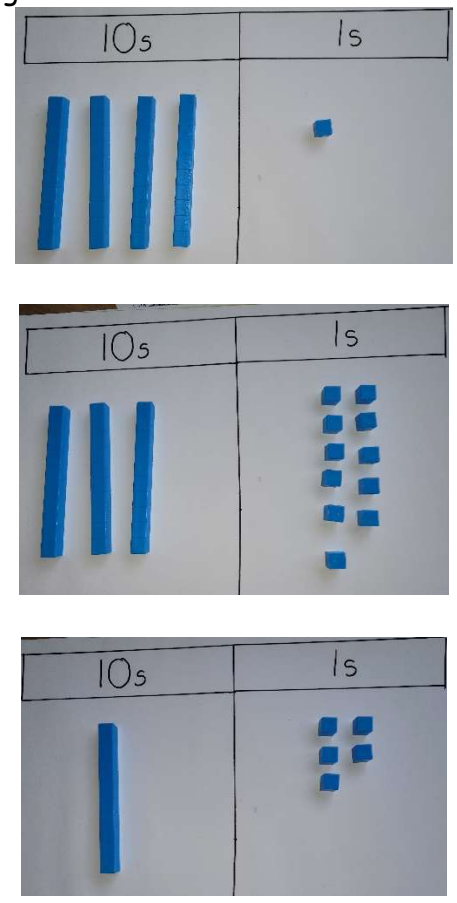
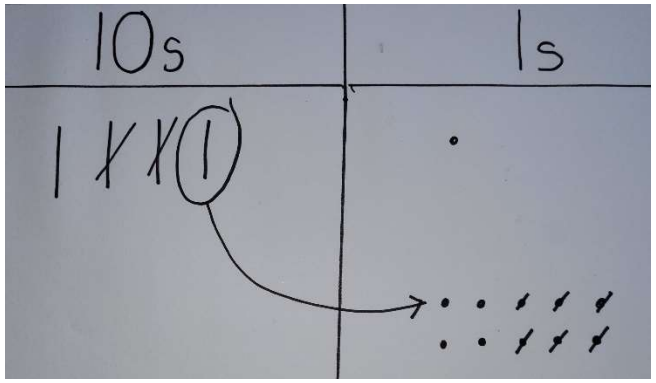
Formal methods should be taught:

- no exchange
- exchange ones to tens
- exchange tens to hundreds
- double exchanging
- larger numbers from Year 4 inc decimals

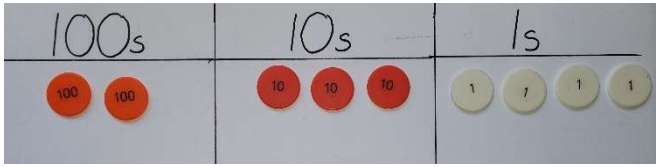
Subtraction

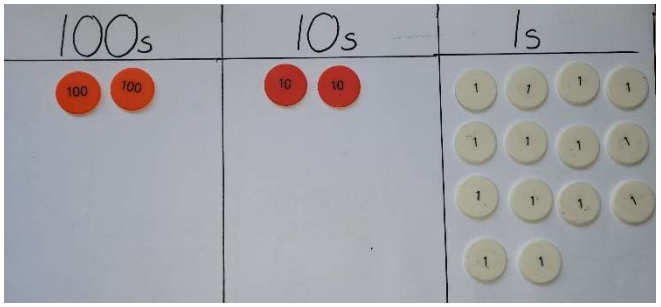
Concrete	Pictorial	Abstract
Physically taking away and removing object from a whole (ten frames, Numicon, cubes and other items such as bean bags could be used). 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = ☐ = 4 – 3 
Counting back (using number lines or number tracks) children start with 6 and count back 2  	Children to represent what they see pictorially. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line. 

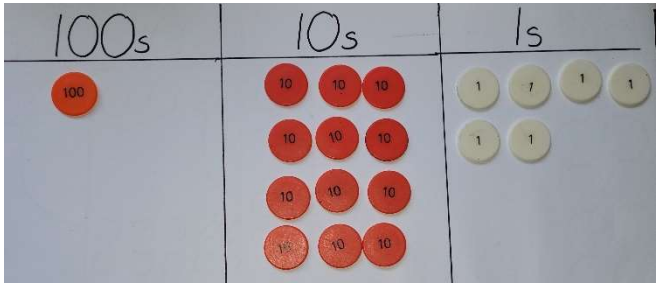
Finding the difference (using cubes, Numicon or Cuisenaire rids, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making 10 using ten frames 14 – 5  - 4  - 1 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend. *(The number that is to be subtracted)* 14 – 5 = 9 / \ 4 1 14 – 4 = 10 10 – 1 = 9

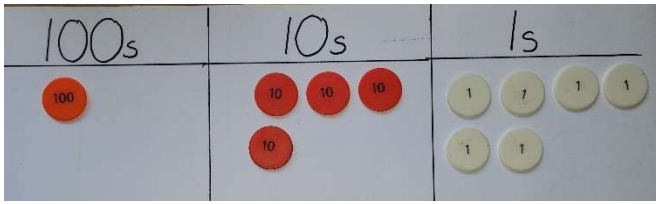
Column Method using base 10. 48 – 7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. Formal Method *(From Year 3)* 4 8 - 7 4 1
Column Method using base 10 and having to exchange. 41 – 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 3 ~~4~~ 1 11 - 2 6 1 5

Column Method using base 10 followed by place value counters. 234 – 88 =



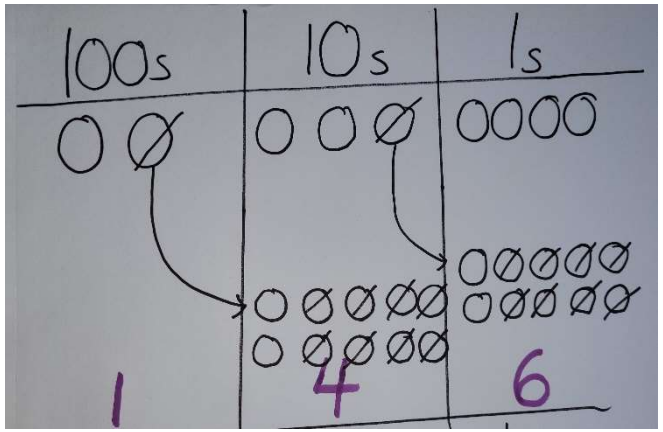






1 **4** **6**

Represent the place value counters pictorially; remembering to show what has been exchanged.



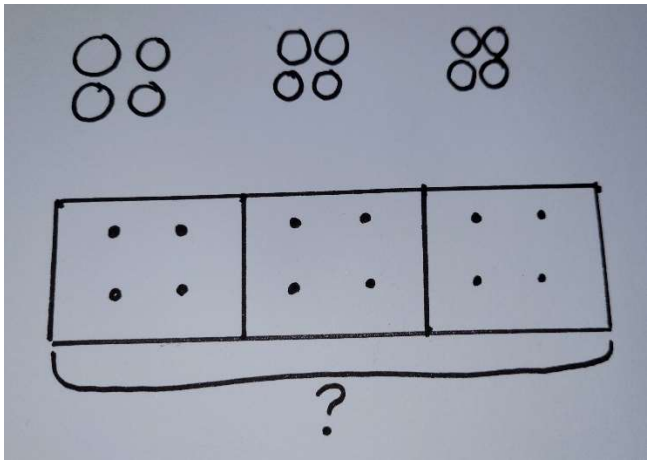
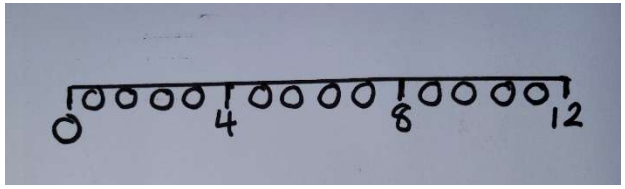
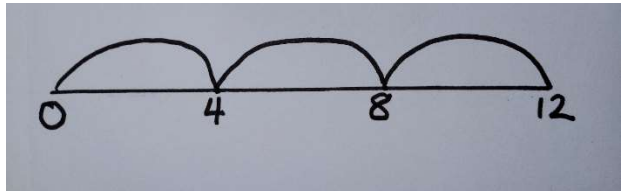
$$\begin{array}{rrrr} & \cancel{1}\,\cancel{2}^{1}\,\cancel{3}^{12}\,4^{1} & & \\ - & & 8 & 8 \\ \hline & 1 & 4 & 6 \end{array}$$

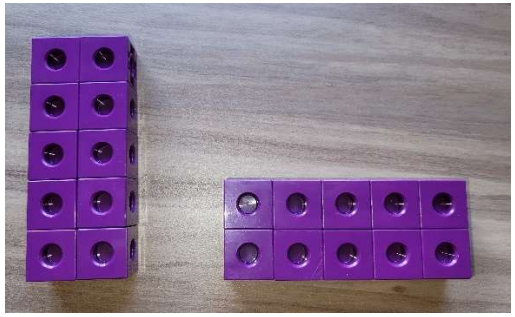
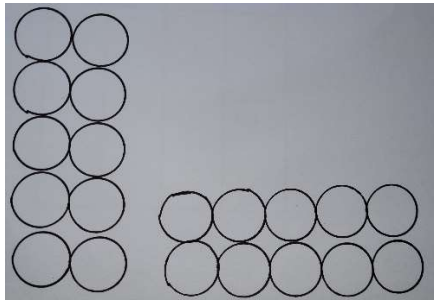
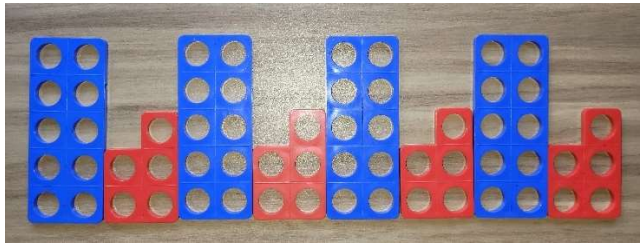
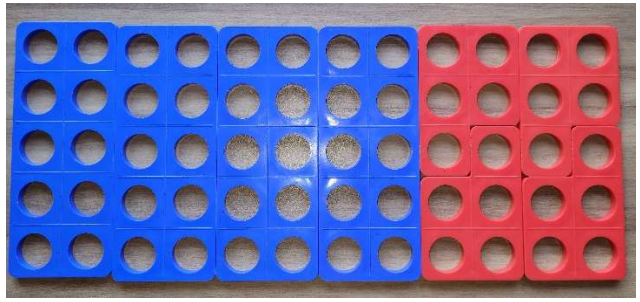
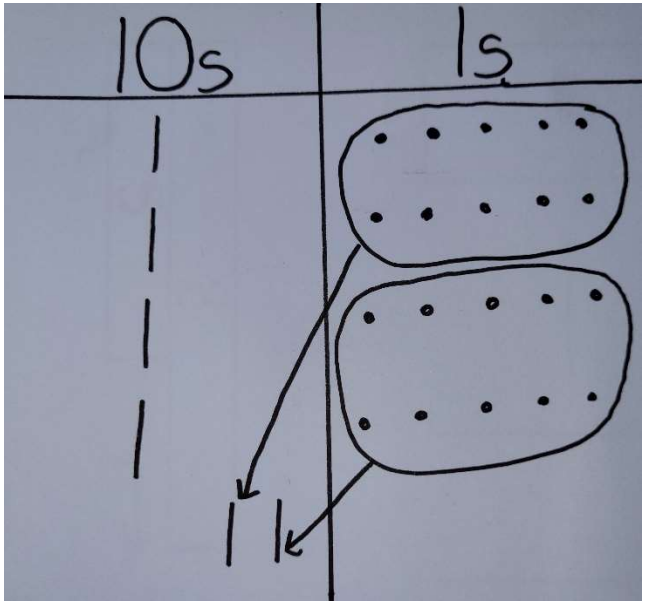
Children must understand what has happened when they have crossed out digits.

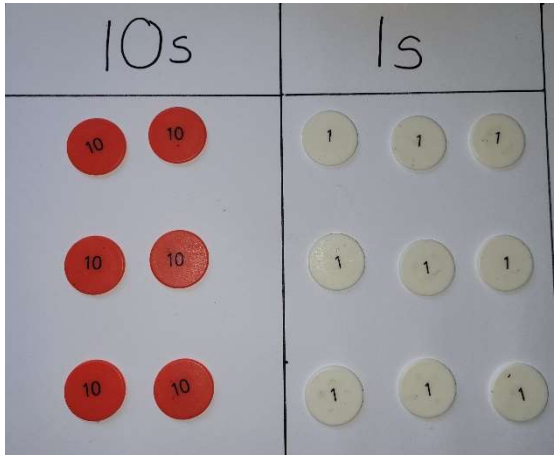
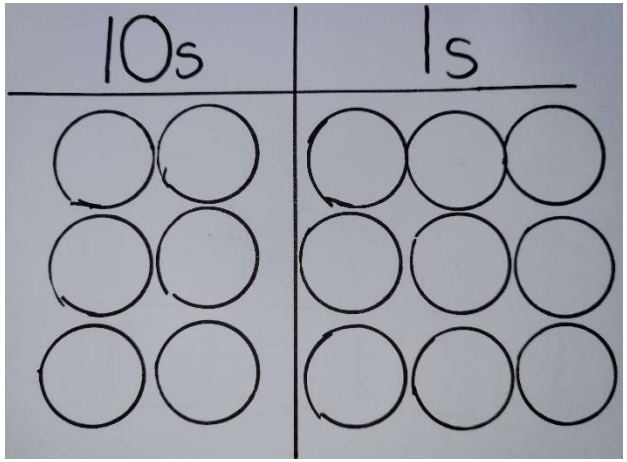
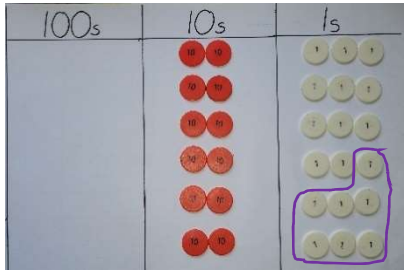
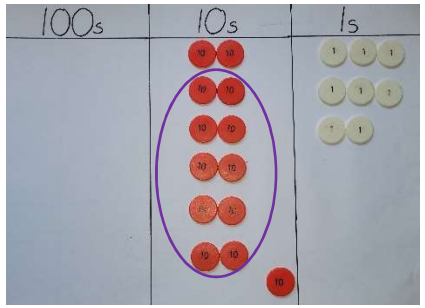
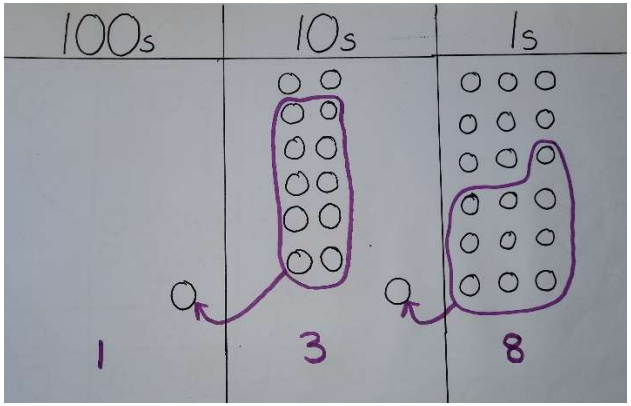
Formal methods should be taught:

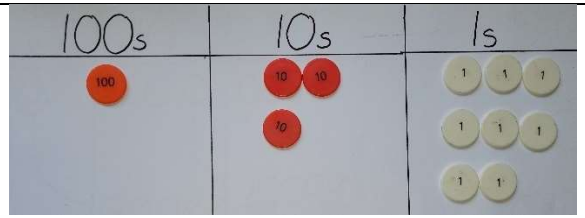
- no exchange
- exchange tens to ones
- exchange hundreds to tens
- double exchanging
- 0 place value holder
- larger numbers from Year 4 including decimals

Multiplication

Concrete	Pictorial	Abstract
Repeated grouping/ repeated addition 3 x 4 4 + 4 + 4 There are 3 equal groups with 4 in each group.  	Children to represent the practical resource in a picture and use a base model. 	3 x 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups 3 x 4 	Represent this pictorially alongside a number e.g. 	Abstract number line showing three jumps of four. 3 x 4 = 12 

Use arrays to illustrate commutativity counters and other objects can also be used. 2 x 5 = 5 x 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 x 5 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 x 15  	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 / \ 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used.

Formal column method with base 10 followed by place value counters. 3 x 23  6 9	Children to represent the counters pictorially.  6 9	Children to record what it is they are doing to show understanding. 3 x 23 3 x 20 = 60 / \ 3 x 3 = 9 20 3 60 + 9 = 69 Formal Method *(From Year 3)* 2 3 x 3 6 9
Formal column method with base 10/ place value counters. 6 x 23   	Children to represent the counters/base 10 pictorially.  	Formal written method 2 3 x 6 1 3 8 *1 1* Formal methods should be taught: - no exchange - exchange ones to tens - exchange tens to hundreds - double exchanging



1 3 8

When children start to multiply 3d x 2d and 4d x 2d etc they should be confident with the abstract.

To get 744 children have solved 6 x 124
To get 2480 children have solved 20 x 124

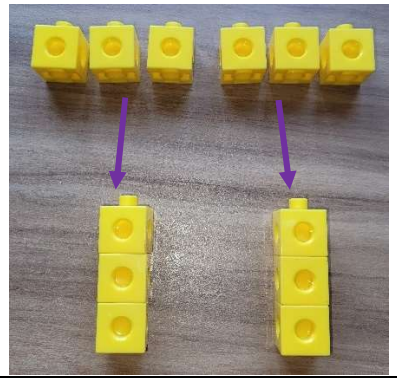
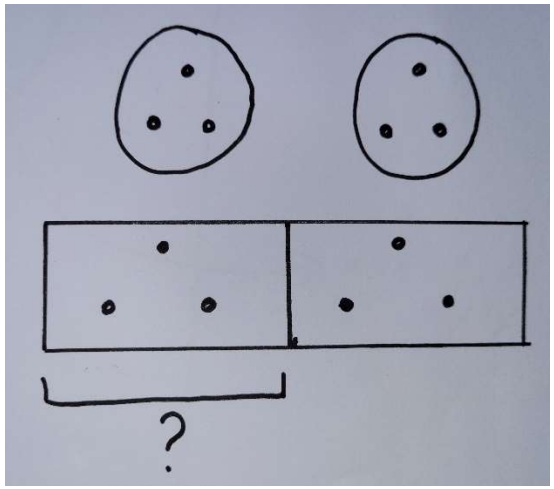
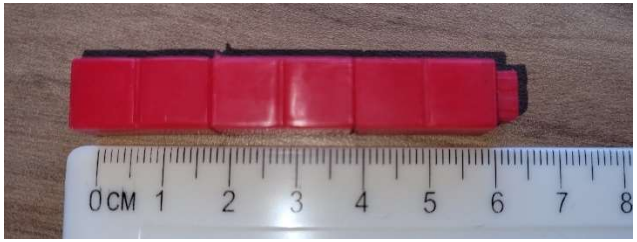
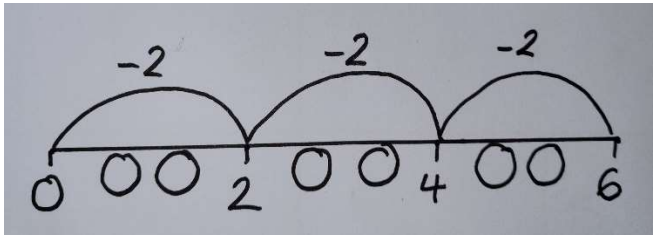
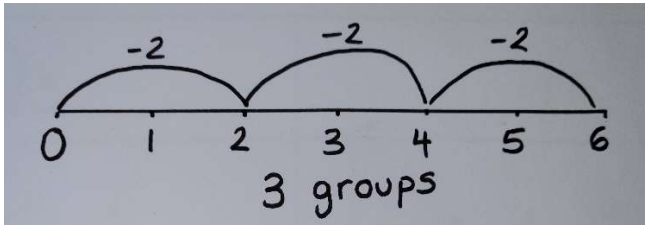
	100	20	4	
20	2000	400	80	= 2480
6	600	120	24	= 744

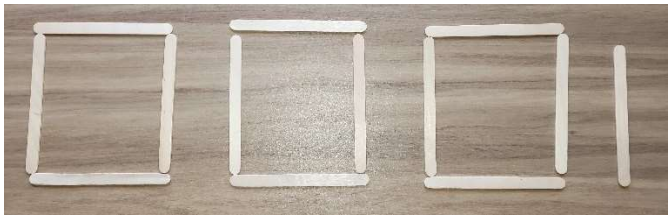
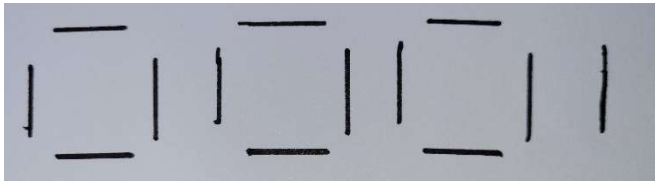
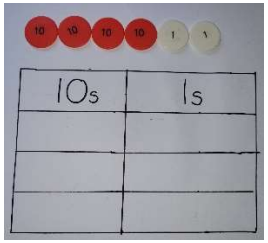
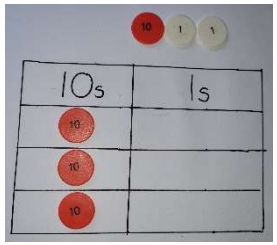
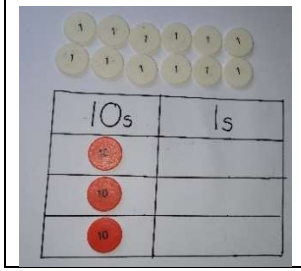
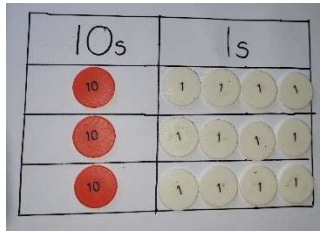
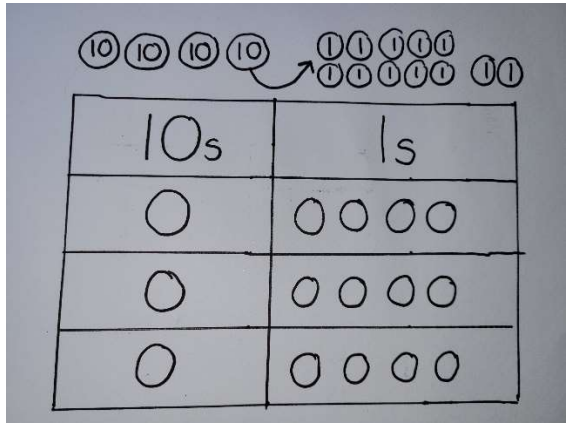
2480 + 744 = 3224

(From Year 5)

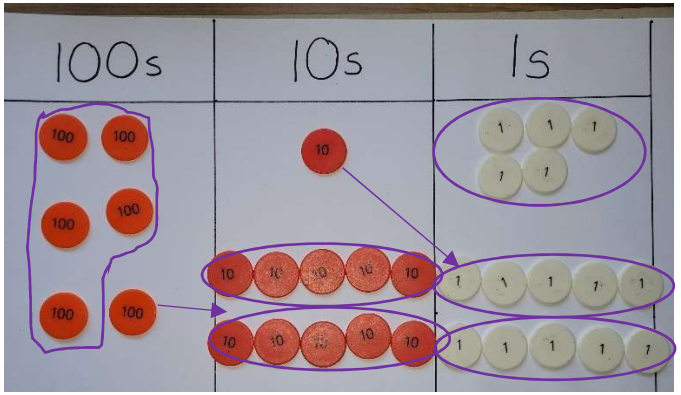
```
    1 2 4
x     2 6
-------
    7 4 4
  1 2
2 4 8 0
-------
3 2 2 4
-------
1 1
```

Division

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2  	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

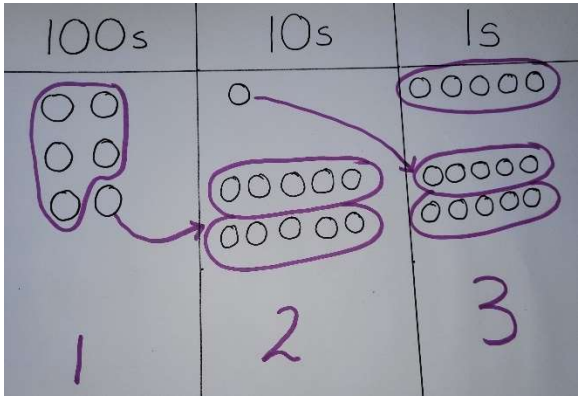
Grouping 2d ÷ 1d with remainders Using lollipops sticks. Cuisenaire rods above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes – squares are made because we are dividing by 4.  There are 3 whole squares with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares with 1 left over.	13 ÷ 4 = 3 reminder 1 Children should be encouraged to use their times tables facts; they could also represent repeated addition on a number line. 3 groups of 4 with 1 left over
Sharing using base 10/place value counters 42 ÷ 3 = 14     	Children to represent the base 10/place value counters pictorially.  	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 = 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group 615 ÷ 5



1 2 3

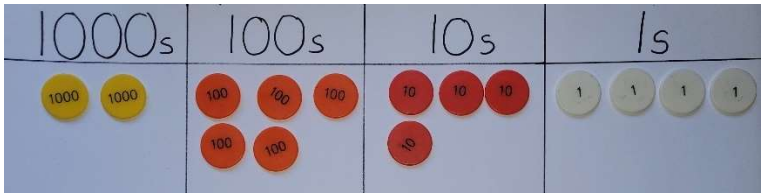
Children to represent the place value counters pictorially.

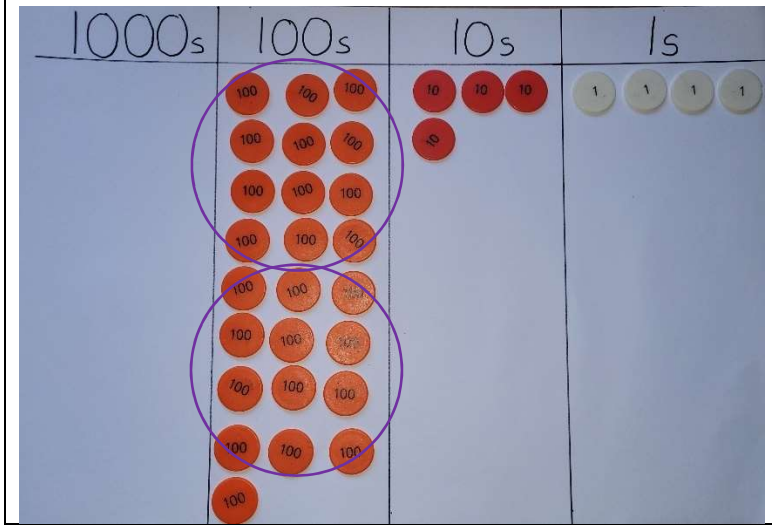


Children to do the calculation using the short division scaffold.

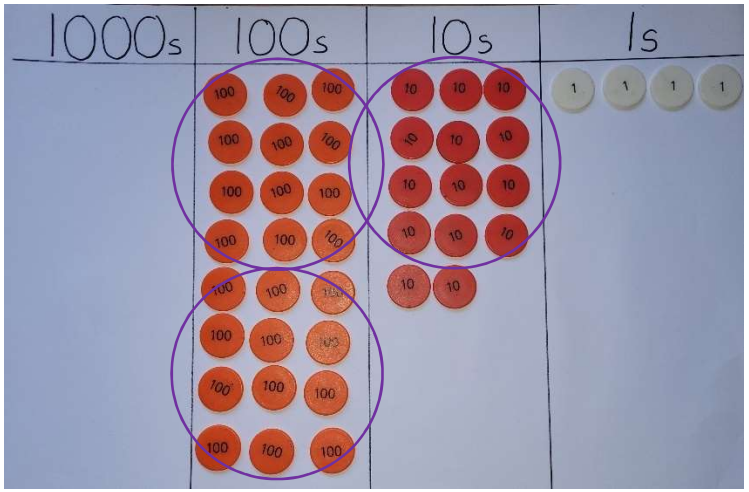
$$\begin{array}{r} 1\ 2\ 3 \\ \hline 5 \,\big|\, 6\ 1\ 5 \\ \quad\ \ 1\ \ 1 \end{array}$$

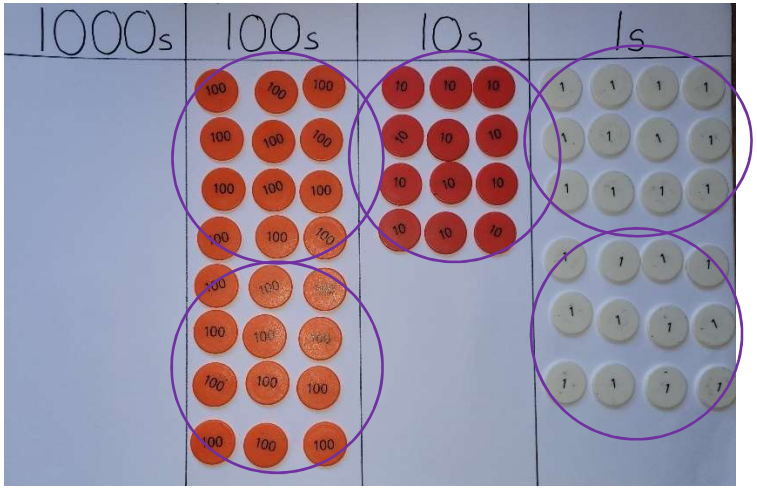
Long division using place value counters to group 2544 ÷ 12





$$\begin{array}{r} 0\ 2\ 1\ 2 \\ \hline 12 \,\big|\, 2\ 5\ 4\ 4 \\ \quad\ \ 2\ \ 1\ \ 2 \\ \quad\ 2\ 4 \\ \hline \quad\ \ 1\ 4 \\ \quad\ \ 1\ 2 \\ \hline \quad\ \ \ \ 2\ 4 \\ \quad\ \ \ \ 2\ 4 \\ \hline \quad\ \ \ \ \ \ 0 \end{array}$$





Remainders to be expressed as decimals and fractions.